M.Sc. (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH05(D) - Optional Paper - Number Theory

P. Pages : 2 **GUG/S/25/13744**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) If (a, m) = 1 then there is an x show that $ax \equiv 1 (\text{mod}\, m)$. Any two such x are congruent (mod m). If $(a, m) > 1$ then there is no such x. **10**

b) State and prove Euler's generalization of Fermat's Theorem. **10**

OR

c) If p is prime, then prove that $(p-1)! \equiv -1 (\text{mod}\, p)$. **10**

d) State and Prove Chinese Remainder Theorem. **10**

UNIT – II

2. a) Solve $x^5 + x^4 + 1 \equiv 0 (\text{mod}\, 3^4)$. **10**

b) Show that congruence $f(x) \equiv 0 (\text{mod}\, p)$ of degree n has at most n solutions. **10**

OR

c) Solve $x^3 + 10x^2 + x + 3 \equiv 0 (\text{mod}\, 3^3)$ **10**

d) Solve $x^2 + x + 7 (\text{mod}\, 81)$ **10**

UNIT – III

3. a) Let p be an odd prime and a is integer relatively prime to p then show that **10**

i) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$

ii) $a \equiv b (\text{mod}\, p) \Rightarrow \left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$

iii) $\left(\frac{1}{p}\right) = 1, \left(-\frac{1}{p}\right) = (-1)^{(p-1)/2}$

b) State and prove Gauss lemma. **10**

OR

c) If p and q are distinct odd primes then show that **10**

$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\{(p-1)/2\}\,\{(q-1)/2\}}$

d) Find all primes p such that $x^2 \equiv 13 \pmod p$ has a solution. **10**

UNIT – IV

4. a) Find all solutions of **10**

$999x - 49y = 5000$

b) Find all solutions of **10**

$97x + 98y = 1000$

OR

c) Find all Pythagorean triples whose terms form an arithmetic progression. **10**

b) For each positive integer n show that $d(n) = \pi(\alpha+1)\ p^{\alpha} \parallel n$ **10**

5. a) If $a \equiv b \pmod m$ and $b \equiv \pmod m$ then show that $a \equiv c \pmod m$. **5**

b) If $d \mid (p-1)$ then show that $x^d \equiv 1 \pmod p$ has d solutions. **5**

c) Let x and y be real numbers, then show that **5**

i) $[x]+[y] \le [x+y] \le [x]+[y]+1$

ii) $-[-x]$ is the least integer $\ge x$.

d) Find all primitive solutions of $x^2 + y^2 = z^2$ having $0 < z < 30$. **5**

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